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Question Booklet No. 900043

DESCRIPTIVE & OBJECTIVE TYPE (MCQ)
SUBJECT : MATHEMATICS

Roll No. | | | | |

Full Marks : 200 (100 Descriptive & 100 MCQ) **Time : 3 Hours**

CANDIDATES SHOULD READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS :

1. The Question Booklet has a seal pasted on it. Candidates should break open the seal only when they are asked to do so by the invigilators.
2. Immediately after breaking open the seal, candidates must check that the Question Booklet contains 100 marks for Section 'A' (Descriptive Type) and 100 marks for Section 'B' (MCQ). If any discrepancy is found, immediately report to the invigilator for changing of the Question Booklet.
3. Candidates must take care to fill up all the required particulars at the appropriate places marked on the Question Booklet as well as on the Answer Booklet. Do not write anything in the spaces provided for office use.
4. For answering Section 'A' questions candidates must answer in Answer Booklet provided.
5. For answering Section 'B' questions candidates must use OMR answer sheet.
   (i) Each question in Section 'B' has 4 (four) alternative answers given as 1, 2, 3, 4 on the OMR answer sheet. Choose the one which you consider to be the best alternative answer and shade the appropriate bubble on the OMR answer sheet.
   (ii) Each question carries 1 (one) mark with no negative marking.
   (iii) Use **only blue** or **black ball point pen** only.

   Correct Method

   ① ● ③ ④

   (iv) The OMR answer sheet will be processed by electronic means using scanner. Hence, any irrelevant/stray marking, incorrect/multiple shadings, faulty erasing of answers or any damage to the OMR answer sheet will be the sole responsibility of the candidate.
6. Page(s) for Rough Work is provided at the end of the Question Booklet.
7. Candidates must hand over the Answer Booklets and OMR answer sheets before leaving the examination hall. They may take away the Question Booklet.
8. Mobile phones and electronics devices are strictly prohibited. Any candidate found in possession of mobile phone in the examination hall will be immediately disqualified and expelled from the examination.
9. Any misconduct or indiscipline in the examination hall/resorting to any form of unfair means/ failure to follow the examination rules will result in disciplinary action as deemed fit by the Commission.
10. The decision of the Commission on all matters is final.

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SEAL

MATHEMATICS
SECTION – A (DESCRIPTIVE)

1. Answer any 2(two) of the following questions. **$(2 \times 20) = 40$ marks**

(i) Using residue theorem evaluate $\oint_c \frac{z^3}{(z-1)(z-2)^2(z-3)} dz$, where c is the circle $|z| = \frac{7}{2}$.

(ii) Discuss the convergence of the series

$$\frac{2}{5}x + \frac{2.4}{5.8}x^2 + \frac{2.4.6}{5.8.11}x^3 + ... \infty$$

(iii) Prove that the n^{th} derivative of $\frac{x^3}{x^2-1}$ for $x = 0$, is zero, when n is even and $-(n!)$, when n is odd and greater than one.

(iv) Find the maximum or minimum value of the function $f(x, y) = x^3 + y^3 - 3axy$.

2. Answer any 2(two) of the following questions. **$(2 \times 10) = 20$ marks**

(i) Find the area bounded by the curve $a^2 x^2 = y^3 (2a - y)$ by double integral.

(ii) Find the directional derivative of $\phi = 5x^2y - 5y^2z = \frac{5}{2}z^2x$ at the point (1, 1, 1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-1}{1}$.

(iii) Prove that "congruence modulo m" is an equivalence relation on the set of all integers.

3. Answer any 8(eight) of the following questions. **$(8 \times 5) = 40$ marks**

(i) Show that the function $f : R \to R$ defined by $f(x) = 5x^3 - 1$ is one-one onto, where R is the set of real numbers.

(ii) Three forces P, Q and R acting on a particle are in equilibrium. If the angle between P and Q is double the angle between P and R, then show that $P = \frac{R^2 - Q^2}{Q}$.

(iii) If sum of two roots of the equation $x^3 - px^2 + qx - r = 0$, is zero, then prove that $pq = r$.

(iv) If $\tan(\alpha + i\beta) = x + iy$, then show that $x^2 + y^2 + 2x \cot 2\alpha = 1$.

(v) Find the inverse Laplace transform of $f(s) = \frac{s^2}{(s^4 - a^4)}$.

(vi) Find the constants a, b and c so that the field

$$F = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k,$$

is irrotational.

(vii) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

(viii) Find solution of the differential equation $\frac{dy}{dx} + 2\tan x . y = \sin x$

(ix) Find the values of a, b and c so that

$$\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2.$$

(x) Find the image of the point (3, –2, 1) in the plane 3x – y + 4z = 2.

SECTION – B OBJECTIVE (MCQ)

1. If $\begin{bmatrix}5 & 4\\1 & 1\end{bmatrix}X=\begin{bmatrix}1 & -2\\1 & 3\end{bmatrix}$, then X is

(1) $\begin{bmatrix}1 & 4\\-2 & 1\end{bmatrix}$ (2) $\begin{bmatrix}3 & -14\\4 & -17\end{bmatrix}$

(3) $\begin{bmatrix}1 & 3\\-2 & 1\end{bmatrix}$ (4) $\begin{bmatrix}-3 & -14\\4 & 17\end{bmatrix}$

2. If two eigen values of $\begin{bmatrix}8 & -6 & 2\\-6 & 7 & -4\\2 & -4 & 3\end{bmatrix}$ are 3 and 15, then the third eigen value is

(1) 1 (2) 2

(3) 0 (4) 5

3. Sum of the binomial coefficient in a binomial expansions $(1 + x)^n$ is

(1) 2^n (2) 0

(3) n^2 (4) n

4. Every polynomial equation of degree n

(1) May have n roots

(2) Has exactly n roots

(3) Has $\frac{n}{2}$ real roots

(4) Has exactly less than n roots

5. If the roots of the equation

$ax^3 + bx^2 + cx + d = 0$,

are in A.P., than

(1) $2b^3 - 9\,abc + 27\,a^2d = 0$

(2) $2b^3 + 9\,abc + 27\,a^2d = 0$

(3) $2a^3 - 9\,abc + 27\,b^2d = 0$

(4) $2a^3 + 9\,abc + 27\,b^2d = 0$

6. The equation whose roots are three times the roots of $x^3 - x^2 + x + 1 = 0$ is

(1) $27x^3 - 27x^2 + 9x + 9 = 0$

(2) $27x^3 - 9x^2 + 3x + 1 = 0$

(3) $x^3 - x^2 + x + 27 = 0$

(4) $x^3 - 3x^2 + 9x + 27 = 0$

7. Expansion of $\cos 6\theta$ in powers of $\cos\theta$ is

(1) $32\cos^6\theta + 48\cos^4\theta - 1$

(2) $32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$

(3) $32\cos^6\theta - 48\cos^4\theta - 18\cos^2\theta + 1$

(4) $48\cos^6\theta - 32\cos^4\theta + 18\cos^2\theta - 1$

8. If $\sin\left(\frac{\pi}{6}+\theta\right)=0.51$, then the approximate value of θ is

(1) 39.7′ (2) 39.7°

(3) 58° (4) 39°

9. If $\cosh(u + iv) = x + iy$, then

(1) $\frac{x^2}{\cosh 2u}+\frac{y^2}{\sinh 2u}=1$

(2) $x^2\sinh^2 u + y^2\cosh^2 u = 1$

(3) $\frac{x^2}{\cosh^2 u}+\frac{y^2}{\sinh^2 u}=1$

(4) $\frac{x^2}{\cosh^2 u}-\frac{y^2}{\sinh^2 u}=1$

10. If Σu_n is a convergence series of positive terms, then $\lim\limits_{n\to\infty} u_n$ is

(1) Finite non-zero

(2) Positive non-zero

(3) 0

(4) ∞

11. If $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=k$, then Σu_n converges is

(1) $k < 1$ (2) $k \leq 1$

(3) $k \geq 1$ (4) $k > 1$

12. The series $\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+ \ldots\ldots \infty$ is

(1) Oscillatory

(2) Convergent

(3) Divergent

(4) None of these

13. The infinite series $\sum \sin\left(\frac{1}{n}\right)$ is

(1) Divergent (2) Convergent

(3) Oscillatory (4) None of these

14. The infinite series $\frac{x}{\sqrt{3}}-\frac{x^2}{\sqrt{5}}+\frac{x^3}{\sqrt{7}} \ldots\ldots \infty$ is

(1) Absolutely convergent for $|x| < 1$

(2) Conditionally convergent for $|x| < 1$

(3) Divergent for $|x| < 1$

(4) Absolutely covergent for $|x| > 1$

15. The series $\sum_{n=0}^{\infty} (2x)^n$ coverges, if

(1) $|x| < 1$ (2) $|x| < 2$

(3) $-\frac{1}{2}<x<\frac{1}{2}$ (4) $-\frac{1}{2}\leq x\leq\frac{1}{2}$

16. A set A contains 5 elements, then the power set of A i.e. P(A) contains

(1) 8 elements (2) 64 elements

(3) 32 elements (4) 16 elements

17. A set A is $A = \{(x, y) / y = e^x, x \in R\}$ and the set B is $B=\{(x,y)/\underset{x\in R}{y=x}\}$, then

(1) $A \subset B$ (2) $A \cap B = \phi$

(3) $B \subset A$ (4) $A \cup B = A$

18. The relation R defined on the set of all integers, defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by n is

(1) an equivalence relation

(2) reflexive relation only

(3) transitive only

(4) None of these

19. The domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are same is

(1) $\left\{2, \frac{-1}{2}\right\}$ (2) $\left\{-2, \frac{1}{2}\right\}$

(3) $\left[2, \frac{2}{-1}\right]$ (4) $\left(-2, \frac{2}{1}\right)$

20. G = {0, 1, 2, 3, 4, 5}, the composition in G is 'addition modulo 6' then the order of 4 is

(1) 5 (2) 4

(3) 2 (4) 3

21. The set N of natural number, with respect to addition and multiplication

(1) is a ring

(2) some times a ring

(3) is commutative ring

(4) is not a ring

22. If W_1 and W_2 are finite-dimensional subspaces of a vector space V, then $W_1 + W_2$ is finite-dimensional and

(1) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2$

(2) $\dim W_1 + \dim W_2 = \dim W_1 . \dim W_2$

(3) $\dim W_1 + \dim W_2 = \dim (W_1 + W_2) + \dim (W_1 \cap W_2)$

(4) $\dim W_1 + \dim W_2 = \dim (W_1 + W_2) - \dim (W_1 \cap W_2)$

23. The real part of $(\sin x + i \cos x)^5$ is

(1) cos 5x (2) sin 5x

(3) –sin 5x (4) –cos 5x

24. If z is a complex number with $|z| = 1$ and $\arg(z) = \frac{3\pi}{4}$, then z is

(1) $\frac{(1-i)}{\sqrt{2}}$ (2) $\frac{(1+i)}{\sqrt{2}}$

(3) $\frac{-(1+i)}{\sqrt{2}}$ (4) $\frac{(-1+i)}{\sqrt{2}}$

25. The value of log(–1) is

(1) $\frac{-\pi}{4}$ (2) $-i\pi$

(3) $i\pi$ (4) π

26. The value of an in the fourier series expansion of $f(x) = x \sin x$, $0 < x < 2\pi$ for $(n \neq 1)$ is

(1) $\frac{2}{n^2-1}$ (2) $\frac{4}{n^2-1}$

(3) $\frac{-2}{n^2-1}$ (4) $\frac{n}{n^2-1}$

27. The n, nth roots of unity form

(1) an A.P. (2) a G.P.

(3) an H.P. (4) None

28. If z_1 and z_2 are two complex numbers, then

(1) $|z_1 + z_2| \leq |z_1| + |z_2|$

(2) $|z_1 + z_2| = |z_1| + |z_2|$

(3) $|z_1 - z_2| = |z_1| - |z_2|$

(4) $|z_1 + z_2| = |z_1| \cdot |z_2|$

29. A function $f(z) = u + iv$ is analytic, then

(1) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y}$

(2) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \ \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

(3) $\frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y}, \ \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

(4) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \ \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$

30. The value of $\int_0^{2+i} (\bar{z})^2 \, dz$ along the line $x = 2y$ is

(1) $\frac{5}{3}(2-i)$ (2) $\frac{5}{3}(2+i)$

(3) $\frac{5}{3}(i-2)$ (4) $\frac{3}{5}(2+i)$

31. The radius of convergence of the series $\sum \frac{1}{2^n+1} . z^n$ is

(1) 1 (2) 2

(3) $\frac{1}{2}$ (4) 2^n

32. Residue of $f(z) = \frac{z+2}{(z+1)^2(z-2)}$ at the pole $z = -1$ is

(1) $\frac{-9}{4}$ (2) $\frac{9}{4}$

(3) $\frac{-4}{9}$ (4) $\frac{4}{9}$

33. The value of $\oint_c \frac{z \, dz}{(z-1)^2(z-2)}$, where is $c : |z-2| = \frac{1}{2}$, is

(1) $\frac{2\pi i}{3}$ (2) $-4\pi i$

(3) $4\pi i$ (4) $\frac{\pi i}{4}$

34. The function y = cosx from $R \to R$ is
(1) One-one onto
(2) Many one into
(3) One-one into
(4) Many one onto

35. The value of $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4}$ is
(1) 0 (2) $\dfrac{1}{4}$
(3) 4 (4) ∞

36. The n^{th} derivative of y = sin(2x + 3) is
(1) $3^n \sin\left(2x+3+\dfrac{n\pi}{2}\right)$
(2) $2^n \sin(2x + 3 + n\pi)$
(3) $2^n \sin\left(2x+3+\dfrac{n\pi}{2}\right)$
(4) $2^n \sin\left(2x+3+\dfrac{n\pi}{3}\right)$

37. The radius of curvature of a circle of radius a is
(1) $\dfrac{1}{a^2}$ (2) $\dfrac{1}{a}$
(3) 2a (4) a

38. If f(x) is continuous in the closed interval [a, b], differentiable in (a, b) and f(a) = f(b), then there exists at least one value c of x in (a, b) such that $f'(c)$ is equal to
(1) $\dfrac{\pi}{4}$ (2) -1
(3) 0 (4) 1

39. The envelope of the family of straight lines $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$, is
(1) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ (2) $\dfrac{a^2}{x^2}+\dfrac{b^2}{y^2}=1$
(3) $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ (4) $\dfrac{a^2}{x^2}-\dfrac{b^2}{y^2}=1$

40. Asymptotes of the curve $x^2y^2 - 9y^2 - 4x^2 = 0$, are
(1) x = –3, y = –2
(2) x = ±3, y = ±2
(3) $x = y^2 - 9$, $y = x^2 - 4$
(4) x = 3, y = 2

41. If $P = k\,v^{3/2}$, then 4% increment in v and 3% increment in *l* requires
(1) 64% increment in P
(2) 7% increment in P
(3) 9% increment in P
(4) 18% increment in P

42. The value of $\lim\limits_{x \to 0} \dfrac{e^{ax}-e^{-ax}}{\log(1+bx)}$, is
(1) 0 (2) $\dfrac{2a}{b}$
(3) $\dfrac{1}{\log(1+b)}$ (4) $\dfrac{2a}{\log(1+b)}$

43. The curve for which the angle between the radius vector and tangent is constant is
(1) $r^2 = a^2 \sin^2\theta$ (2) $r = a\sin\theta$
(3) $r = a\cos\theta$ (4) $r = a\,e^{b\theta}$

44. The value of $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is
(1) $\dfrac{\pi}{2}$ (2) $\dfrac{\pi}{4}$
(3) 1 (4) 0

45. The area bounded by the parabola $y^2 = 4ax$ its latusrectum is
(1) $\dfrac{2a^2}{3}$ (2) $\dfrac{4a^2}{3}$
(3) $\dfrac{a^2}{3}$ (4) $\dfrac{8a^2}{3}$

46. The entire length of the cardioid $r = a(1 + \cos\theta)$ is

(1) 2a (2) 8a

(3) 4a (4) $\frac{8a}{3}$

47. Volume of the solid formed by the revalution of the area of the loop of the curve $y^2(a + x) = x^2(3a - x)$ about x-axis is

(1) $\pi a^3 (8 \log 2 - 3)$

(2) $8a^3 (\log 2 - 3)$

(3) $\pi a^3 (8 \log 3 - 2)$

(4) $\frac{4}{3}\pi a^3$

48. Area of the region bounded by y-axis and the curves y = sinx, y = cosx in the first quadrant is

(1) $\sqrt{3}-1$ (2) $\sqrt{2}$

(3) $\sqrt{2}-1$ (4) $\sqrt{2}+1$

49. Area of a loop of the curve $r = a \sin 3\theta$, is

(1) $\frac{\pi a^2}{12}$ (2) $\frac{\pi a^2}{9}$

(3) $\frac{\pi a^2}{6}$ (4) $\frac{\pi a^2}{4}$

50. The differential equation for

$$y = e^x (A \cos x + B \sin x)$$

is

(1) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$

(2) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}-2y=0$

(3) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$

(4) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

51. Solution of $x^2y\,dx - (x^3 + y^3)\,dy = 0$ is

(1) $cx = e^{\frac{x^3}{3y^3}}$ (2) $cy = e^{\frac{x^3}{3y^3}}$

(3) $cy = e^{\frac{y^3}{3x^3}}$ (4) $cx = e^{\frac{y^3}{3x^3}}$

52. Solution of $(x+1)\frac{dy}{dx} - ye^{3x}(x+1)^2$ is

(1) $y = c(x + 1)\, e^{3x+1}$

(2) $y = c(x + 1)\, e^{3x}$

(3) $y=\left(\frac{1}{3}e^{3x}+c\right)(x+1)$

(4) $y = (e^{3x} + c)\,(x + 1)$

53. Solution of

$(y^2 e^{xy^2} + 4x^3)dx + (2xy\, e^{x}y^2 - 3y^2)dy = 0$, is

(1) $x^3 + y^4 - e^{xy^2} = c$

(2) $x^4 - y^3 + e^{xy^2} = c$

(3) $y^3 + 4x^3 - e^{xy^2} = c$

(4) $y^4 - x^3 + e^{xy^2} = c$

54. Solution of $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=0$, given $y(0) = 0$, $\frac{dy}{dx}(0)=15$, is

(1) $15\,(e^{2x} - e^{3x})$

(2) $5\,(e^{-2x} + e^{-3x})$

(3) $15\,(e^{-2x} - e^{-3x})$

(4) $5\,(e^{2x} - e^{-3x})$

55. Particular integral of

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4y=e^x \cos x,$$

is

(1) $\frac{e^x \cos x}{3}$ (2) $\frac{e^x \cos x}{4}$

(3) $\frac{e^x \sin x}{2}$ (4) $\frac{e^x \cos x}{2}$

56. The value of $\underset{\substack{x \to \infty \\ y \to 2}}{\text{Lt}} \dfrac{xy+1}{x^2+2y^2}$ is

(1) 0 (2) $\frac{1}{2}$

(3) 1 (4) ∞

57. If $u = \sin^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

(1) $\frac{1}{2}\cos 2u$ (2) $\frac{1}{2}\tan u$

(3) $\frac{1}{2}\cot u$ (4) $\sin 2u$

58. The equation of tangent plane to the surface $z^2 = 4(1 + x^2 + y^2)$ at the point (2, 2, 6) is

(1) $4x + 4y + 3z + 2 = 0$

(2) $4x + 4y - 3z - 2 = 0$

(3) $4x + 4y - 3z + 2 = 0$

(4) $4x + 4y + 3z - 2 = 0$

59. The % error in measuring the area of an ellipse when there is one percent error in measuring the major and minor axes is

(1) 3.14 (2) 0

(3) 1 (4) 2

60. The stationary points of the function $f(x, y) = x^3 y^2(1 - x - y)$ is

(1) (3, 2) (2) $\left(\frac{1}{2}, \frac{1}{3}\right)$

(3) $\left(\frac{1}{3}, \frac{1}{2}\right)$ (4) (2, 3)

61. The value of $\int_0^1 \int_0^{1-x} dy\, dx$ is

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$

(3) 1 (4) $\frac{1}{2}$

62. On changing to polar coordinates $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} dy\, dx$, becomes

(1) $\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{2a\cos\theta} r\, dr\, d\theta$

(2) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_{r=0}^{2a\cos\theta} r\, dr\, d\theta$

(3) $\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{2a\cos\theta} dr\, d\theta$

(4) $\int_{\theta=\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_{r=0}^{2a\cos\theta} dr\, d\theta$

63. The value of $\iint_R x^2y^3 dx\, dy$, where $R : 0 \le x \le 1,\ 0 \le y \le 3$, is

(1) $\frac{81}{8}$ (2) $\frac{81}{4}$

(3) $\frac{27}{4}$ (4) $\frac{27}{8}$

64. The value of

$$\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z)\, dy\, dx\, dz$$

is

(1) 0 (2) $\frac{1}{8}$

(3) $\frac{9}{8}$ (4) $\frac{1}{27}$

65. The value of $\int_0^\infty \int_x^\infty \dfrac{e^{-y}}{y} dy\, dx$ is

(1) $-\infty$ (2) ∞

(3) 1 (4) 0

66. Projection of the vector $i - 2j + k$ on $4i - 4j + 7k$ is

(1) $\frac{9}{19}$ (2) $\frac{19}{9}$

(3) 19 (4) $\frac{1}{9}$

67. Angle between any two diagonals of a cube is

(1) $\cos^{-1}\left(\frac{1}{3}\right)$ (2) $\cos^{-1}\left(\frac{2}{3}\right)$

(3) $\cos^{-1}\left(\frac{1}{2}\right)$ (4) $\cos^{-1}\left(\frac{3}{4}\right)$

68. The volume of the parallelopiped whose edges are given by the vectors $A = 2i - 3j + 4k$, $B = i + 2j - k$ and $c = 3i - j + 2k$, is

(1) 49 (2) 14

(3) 15 (4) 7

69. If $\vec{F}$ is a conservative force then

(1) grad $\vec{F} = 0$

(2) curl $\vec{F} = 0$

(3) div $\vec{F} = 0$

(4) grad (curl $\vec{F}$) $\neq 0$

70. The angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2) is

(1) $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$

(2) $\cos^{-1}\left(\frac{3}{8\sqrt{21}}\right)$

(3) $\cos^{-1}\left(\frac{1}{3}\right)$

(4) $\cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$

71. If C is a simple closed curve in the xy-plane not containing the origin and $\vec{F} = \frac{yi - xj}{x^2 + y^2}$ then the value of $\int_C \vec{F} \cdot dr$ is

(1) $\frac{3}{4}$ (2) 4

(3) zero (4) $\frac{1}{2}$

72. The value of λ for which the vector $(x + 3y)i + (y - 2z)j + (x + \lambda z)k$ is a solenoidal vector is

(1) 3 (2) 2

(3) –2 (4) –1

73. The work done in moving a particle in the force field $\vec{F} = 3x^2 i + (2xz - y)j + zk$ along the straight line from (0,0,0) to (2,1,3) is

(1) 9 (2) 8

(3) 18 (4) 16

74. The scaler potential function ϕ for the conservative force $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is

(1) $\phi = xz^3 + y^3 \cos x + 4y + 2z$

(2) $\phi = xz^3 + y^2 \sin x - 4y + 2z$

(3) $\phi = xz^3 - y^2 \sin x - 4y + 2z$

(4) $\phi = xz^3 + y^2 \cos x + 4y + 2z$

75. If $\vec{r}$ is the position vector of any point, then div $\vec{r}$ is

(1) 0 (2) $\frac{1}{3}$

(3) 2 (4) 3

76. The equation of a line which passes through the point of intersection of the straight lines $x + 2y = 5$ and $3x + 7y = 17$ and is perpendicular to the line $3x + 4y = 10$ is

(1) $4x - 3y - 2 = 0$

(2) $4x - 3y + 2 = 0$

(3) $4x + 3y - 2 = 0$

(4) $4x + 3y + 2 = 0$

77. If the lines ax + 1 2y + 1 = 0, bx + 1 3y + 1 = 0 and cx + 1 4y + 1 = 0 are concurrent, then a, b, c are in

(1) H.P. (2) A.P.

(3) G.P. (4) None

78. Distance of the point (2, 3) form line $2x - 3y + 9 = 0$, measured along the line $x - y + 1 = 0$ is

(1) $4\sqrt{2}$ (2) $2\sqrt{2}$

(3) $3\sqrt{2}$ (4) 3

79. Equation of the circle formed by the coordinate axes and the line $4x + 3y - 6 = 0$ is

(1) $x^2 + y^2 + 6x + 6y - 9 = 0$

(2) $x^2 + y^2 + 6x + 6y + 9 = 0$

(3) $x^2 + y^2 - 6x - 6y - 9 = 0$

(4) $x^2 + y^2 - 6x - 6y + 9 = 0$

80. The coordinates of the focus of the parabola $y^2 - x - 2y + 2 = 0$ are

(1) $\left(\frac{1}{4}, 0\right)$ (2) (2, 1)

(3) $\left(\frac{5}{4}, 1\right)$ (4) (–1, –2)

81. The coordinates of the point of intersection of the line $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z+2}{-2}$ with the plane $3x + 4y + 5z = 5$ are

(1) (5, 15, –14) (2) (1, 2, 3)

(3) (3, 4, 5) (4) (–1, –3, –2)

82. The distance between the planes $2x + 2y + z = 6$ and $4x + 4y + 2z = 7$ is

(1) $\frac{1}{3}$ (2) $\frac{5}{6}$

(3) $\frac{5}{3}$ (4) $\frac{13}{6}$

83. The equation of a plane through the point (2, –3, 1) and parallel to the plane $3x - 4y + 2z = 6$ is

(1) $3x - 4y + 2z - 20 = 0$

(2) $3x - 4y + 2z + 20 = 0$

(3) $3x - 4y + 2z - 18 = 0$

(4) $-3x + 4y - 2z = -6$

84. The angle between the planes $x - y + 2z - 9 = 0$ and $2x + y + z - 7 = 0$ is

(1) 45° (2) 30°

(3) 90° (4) 60°

85. The equation of the sphere which touches the plane $x - 2y - 2z = 7$ at the point (3, –1, –1) and passes through the point (1, 1, –3) is

(1) $x^2 + y^2 + z^2 + 10y + 10z - 31 = 0$

(2) $x^2 + y^2 + z^2 - 10y - 10z + 31 = 0$

(3) $x^2 + y^2 + z^2 - 10y - 10z - 31 = 0$

(4) $x^2 + y^2 + z^2 - 10x - 10y - 9 = 0$

86. The equation of the right circular cone generated when the line $2y + 3z = 6$, $x = 0$ revolves about z-axis is

(1) $4x^2 + 4z^2 + 9y^2 + 36x - 36 = 0$

(2) $4x^2 + 4z^2 - 9y^2 + 36z - 36 = 0$

(3) $4x^2 + 4y^2 + 9z^2 + 36z - 36 = 0$

(4) $4x^2 + 4y^2 - 9z^2 + 36z - 36 = 0$

87. Forces M and N are acting at a point O. Angle between M and N is 150°. Their resultant acts at O, has magnitude 2 units and is perpendicular to M, then the magnitudes of M and N are

(1) $M = 2, \quad N = 4\sqrt{3}$

(2) $M = 4, \quad N = 2\sqrt{3}$

(3) $M = 2\sqrt{3}, \quad N = 2$

(4) $M = 2\sqrt{3}, \quad N = 4$

88. Two men carry a weight of 240 newtons by means of two ropes fined to the weight. One rope is inclined at 60° to the vertical and the other at 30°. Then the tensions in the ropes are

(1) $T_1 = 120$ N, $T_2 = 120$ N

(2) $T_1 = \frac{120}{\sqrt{2}}$ N, $T_2 = 120\frac{\sqrt{3}}{2}$ N

(3) $T_1 = 120$ N, $T_2 = 120\sqrt{3}$ N

(4) $T_2 = 120$ N, $T_1 = 120\sqrt{3}$ N

89. Three parallel forces P, Q, R act three points A, B and C of a rod at distances of 2m, 8m and 6m respectively from one end. If the rod is in equilibrium then P : Q : R is

(1) 1 : 2 : 3 (2) 1 : 3 : 2

(3) 3 : 1 : 2 (4) 3 : 2 : 1

90. The forces respresented in magnitude, direction and position by the sides of a triangle taken in order

(1) reduce to a single force

(2) are in equilibrium

(3) are never in equilibrium

(4) form a couple

91. A bullet fired into a target loses half of its velocity in penetrating 3 cm. The bullet will penetrate further by

(1) 3 cm (2) 1 cm

(3) 2 cm (4) $\frac{3}{2}$cm

92. Two bodies of different masses m_1 and m_2 are dropped from different heights h_1 and h_2. The ratio of the times taken by the two bodies to reach the ground is

(1) $\sqrt{h_1} : \sqrt{h_2}$ (2) $h_1 : h_2$

(3) $h_2 : h_1$ (4) $h_1^2 : h_2^2$

93. A gun can fire with a velocity u in all directions from a given position an a horizontal plane. The shots will fall on the plane within a circle of radius

(1) $\frac{u^2}{2g}$ (2) $\frac{2u^2}{g}$

(3) $\frac{u^2}{g}$ (4) $\frac{u^2}{3g}$

94. A cricket ball of mass 200 gms moving with a velocity of 20 m/sec, is brought to rest by a player in 0.1 sec. The average force applied by the player is

(1) 40 Newtons (2) 4 Newtons

(3) 10 Newtons (4) 20 Newtons

95. Laplace transform of $f(t) = \sqrt{t}\, e^{2t}$ is

(1) $\frac{2}{\sqrt{\pi}} \frac{1}{(s+2)^{3/2}}$

(2) $\frac{2}{\sqrt{\pi}} \frac{1}{(s-2)^{3/2}}$

(3) $\frac{\sqrt{\pi}}{2} \frac{1}{(s-2)^{3/2}}$

(4) $\frac{\sqrt{\pi}}{2} \frac{1}{(s+2)^{3/2}}$

96. Laplace transform of $f(t) = \left(\frac{1-e^t}{t}\right)$ is

(1) $\log\left(\frac{s-1}{s}\right)$

(2) $\log\left(\frac{s+1}{s}\right)$

(3) $\log\left(\frac{s}{s-1}\right)$

(4) $\log\left(\frac{s}{s+1}\right)$

97. The value of $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t} dt$ is

(1) $\log\left(\frac{b}{a}\right)$ (2) $\log\left(\frac{a}{b}\right)$

(3) $\log(a + b)$ (4) $\log(a - b)$

98. The inverse laplace transform of $\log\left(\frac{s+1}{s-1}\right)$ is

(1) $t(e^t + e^{-t})$ (2) $\frac{2\sinh t}{t}$

(3) $2t \sinh t$ (4) $t(e^t - e^{-t})$

99. Inverse laplace transform of $\frac{se^{-s/2}}{s^2+\pi^2}$ is

(1) $\sin\frac{\pi t}{2} U\left(t-\frac{1}{2}\right)$

(2) $\cos\frac{\pi t}{2} U\left(t-\frac{1}{2}\right)$

(3) $\cos \pi t\, U\left(t-\frac{1}{2}\right)$

(4) $\sin \pi t\, U\left(t-\frac{1}{2}\right)$

100. If $f(t) = 3$, $0 < t < 2$ and $f(t)$ is a periodic function of period $= 0$, $2 < t < 4$, 4, then laplace transform of $f(t)$ is

(1) $\left(\frac{4}{1-e^{-4s}}\right)$ (2) $\frac{3}{s(1+e^{-2s})}$

(3) $\frac{4}{s(1+e^{-2s})}$ (4) $\frac{1}{(1-e^{-4s})}$